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The Susceptance of a Circular Obstacle to an Incident Dominant Circular-Electric Wave

by

L. S. Sheingold

June 15, 1952

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The Susceptance of a Circular Obstacle to an Incident Dominant Circular-Electric Wave

by

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Abstract

The problem of the susceptance of a circular obstacle in a circular waveguide with an incident TE_{01} mode is solved by a variational method. Theoretical expressions are obtained which are in good agreement with experimental results. Curves of normalized susceptance as a function of relative aperture, guide wavelength, and free-space wavelength are included.

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I

Introduction

Recent interest in the practical transmission of millimeter waves has created a demand for a practical low-loss transmission line. The dominant circular-electric mode is ideally suited for this application. During the past few years several researchers have investigated the anomolous attenuation character of this mode and have considered its practical application.[1-6] The problem of exciting a relatively pure TE_{01} mode with sufficiently high available power has been solved satisfactorily by Southworth and his coworkers at the Bell Telephone Laboratories.[7,8] Theoretical and experimental investigations of attenuation in circular waveguide indicate that the TE_{01} mode can propagate with negligible additional attenuation due to mode conversion caused by slight random variations in the diameter of standard tubing.[9,10] The theoretical problem of a curved circular waveguide has been considered in great detail.[11,12] The complication arises from the fact that the TE_{01} mode is not the dominant mode in the circular waveguide. In particular the TE_{01} mode has the same phase velocity as the TM_{11} mode, and therefore any major disturbance such as a curvature of the waveguide axis will convert the energy of the TE_{01} mode to the TM_{11} mode as well as to other propagating modes. Only uniform circular waveguides are considered here because to date this is the only practical method of transmission of the TE_{01} mode.

This paper is concerned with the determination of the susceptance of a symmetrical obstacle to an incident dominant circular-electric wave. The obstacle shown in Fig. 1 is assumed to possess infinite conductivity and negligible thickness. The obstacle is planar and has circular symmetry. It is assumed that the exciting mode is the dominant circular-electric mode; hence the symmetry of the obstacle limits the higher attenuating modes excited by the obstacle discontinuity to the higher circular-electric modes. The energy stored in the higher modes is entirely magnetic. The circular obstacle can accordingly be represented as a shunt inductance on the analogous transmission line.

The general procedure for the determination of the normalized susceptance of a thin obstacle by a variational principle has been described in several references.[13-18] The accuracy of the obtained theoretical results depends on the nature of the assumed trial field. It has been found that the laborious calculations required to evaluate the normalized susceptance usually limit the trial function to a series of two terms.

In this report the problem of the circular obstacle is

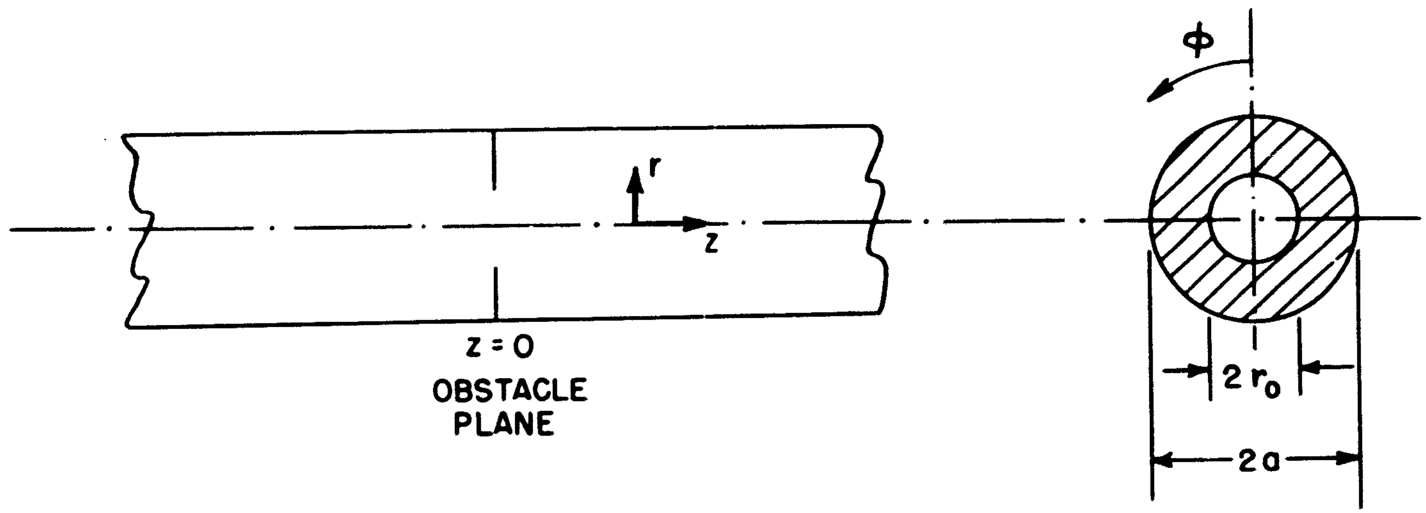


FIG. I OBSTACLE IN CIRCULAR WAVEGUIDE

considered from two aspects, one involving a lower-bound variational formulation and the other an upper-bound variational formulation.

An expression which is a lower bound to the true value of the obstacle susceptance is expressed in terms of the tangential magnetic field (or current) on the obstacle. The chosen trial function consists of two terms: the first is the unperturbed current distribution on an aperture-less obstacle; the second is a correction term. This lower-bound expression yields susceptances which are accurate for extremely small apertures.

The normalized susceptance of the circular obstacle which serves as an upper bound to the true value of the susceptance is a function of the tangential electric field in the obstacle aperture. Two different trial functions are used in the upper-bound formulation. The first assumed trial function is a series of two terms consisting of the electric field in the aperture as the radius of the aperture approaches that of the waveguide, and a correction term. Theoretical values for the susceptance obtained by this approach yield accurate results for obstacles with large apertures. The second trial function consists of a two-term trial function for the aperture electric field and is chosen to satisfy the boundary conditions at the obstacle discontinuity in the proper fashion.

Theoretical results obtained from the lower- and upper-bound expressions are in excellent agreement with the experimental results.

II

Variational Formulation

The electromagnetic nature of the obstacle problem is simplified by representing the discontinuity as a lumped constant equivalent parameter in a TE_{01} transmission line. This is accomplished by expressing the field components in terms of mode voltages and mode currents. Owing to the symmetrical excitation and the symmetry of the obstacle, the modes of interest are the circular-electric modes. For harmonic time dependence ($\exp(j\omega t)$) the field components of the circular-electric modes expressed as a function of mode voltages and currents are:

$$E_r = 0$$

$$E_\phi = \frac{J_1(\rho_n r/a)}{\sqrt{\pi}\, a\, J_o(\rho_n)} V_n(z) = e_n(r)\, V_n(z)$$

$$E_z = 0$$

$$H_r = \frac{-J_1(\rho_n r/a)}{\sqrt{\pi}\, a\, J_o(\rho_n)} I_n(z) = h_n(r)\, I_n(z) \qquad (1)$$

$$H_\phi = 0$$

$$H_z = \frac{-j\eta\rho_n J_o(\rho_n r/a)}{ka^2\sqrt{\pi}\, J_o(\rho_n)} V_n(z)$$

$$e_n(r) = -h_n(r)$$

A tabulation of the necessary quantities is given below.

r_o = obstacle aperture radius

a = radius of the circular waveguide

$\delta = \frac{r_o}{a}$ = relative aperture or aperture radius to guide radius ratio

$k = \omega\sqrt{\mu\varepsilon}$ = free-space wave number

ρ_n = nth root of $J_1(x) = 0$

$\beta_n = \sqrt{k^2-(\frac{\rho_n}{a})^2}$ = wave number of guide

$\eta = \sqrt{\frac{\varepsilon}{\mu}}$ = intrinsic admittance of free space

$\zeta = \frac{1}{\eta} = \sqrt{\frac{\mu}{\varepsilon}}$ = intrinsic impedance of free space

Substitution of the field components of equation (1) into Maxwell's field equations results in two simultaneous equations which are identical in form to the conventional transmission-line equations.

$$\frac{dV_n(z)}{dz} = -jk\zeta I_n(z) = -j\beta_n Z_{cn} I_n(z) \qquad (2a)$$

$$\frac{dI_n(z)}{dz} = \frac{-j\beta_n^2}{k}\eta V_n(z) = -j\beta_n Y_{cn} V_n(z) \qquad (2b)$$

Using the transmission-line analog, it is possible to define a characteristic admittance and a characteristic impedance for each circular-electric transmission line. For propagating modes the characteristic admittance Y_{cn} and the characteristic impedance Z_{cn} are real quantities and related by

$$Y_{cn} = \frac{1}{Z_{cn}} = \frac{\sqrt{k^2 - (\frac{\rho_n}{a})^2}}{\omega\mu} \qquad (3)$$

while for nonpropagating modes Y_{cn} and Z_{cn} are imaginary. Therefore

$$Y_{cn} = \frac{1}{Z_{cn}} = \frac{-j\sqrt{(\frac{\rho_n}{a})^2 - k^2}}{\omega\mu} \qquad (4)$$

The thin circular obstacle can be represented as an equivalent shunt susceptance B on the TE_{01} transmission line. The TE_{01} field exciting the obstacle is arbitrary, making it possible to separate the exciting field into even and odd components. The even case requires the tangential magnetic field to vanish in the aperture while the tangential electric field is at its maximum value. Odd excitation requires the tangential electric field to vanish in the aperture and the tangential magnetic field to be maximum. Considering only even excitation and applying[9] the bisection theorem immediately leads to

$$Y_{in} = -j\,\frac{B}{2}\,, \qquad (5)$$

where

Y_{in} is the admittance looking into the bisected structure (see Fig. 2).

The obstacle susceptance can now be determined by solving Maxwell's field equations in the region to the left of the obstacle with the following boundary conditions

$$\begin{aligned} E_\phi &= 0 \qquad r = a \qquad -\infty < z < 0 \\ E_\phi &= 0 \qquad r_o < r < a \qquad z = 0 \\ H_r &= 0 \qquad 0 < r < r_o \qquad z = 0 \end{aligned} \tag{6}$$

The problem can be formulated in terms of the tangential electric field in the aperture or the tangential magnetic field on the surface of the obstacle.

The uniform field structure in the ϕ-direction permits the transverse electric field to be expressed as a sum of mode voltages of the circular-electric type. A similar representation in terms of the mode currents can be obtained for the transverse magnetic field. From equation (1) these quantities are given by

$$E_t = E_\phi(r,z) = \sum_{n=1}^{\infty} e_n(r)V_n(z) \tag{7a}$$

$$H_t = H_r(r,z) = \sum_{n=1}^{\infty} h_n(r)I_n(z) \tag{7b}$$

From orthogonality conditions the mode voltage amplitudes and mode current amplitudes can be expressed in the form of integrals over the circular guide cross section, resulting in

$$V_n(z) = \int_S E_\phi(r,z)e_n(r)d\sigma \tag{8a}$$

$$I_n(z) = \int_S H_\phi(r,z)h_n(r)d\sigma \tag{8b}$$

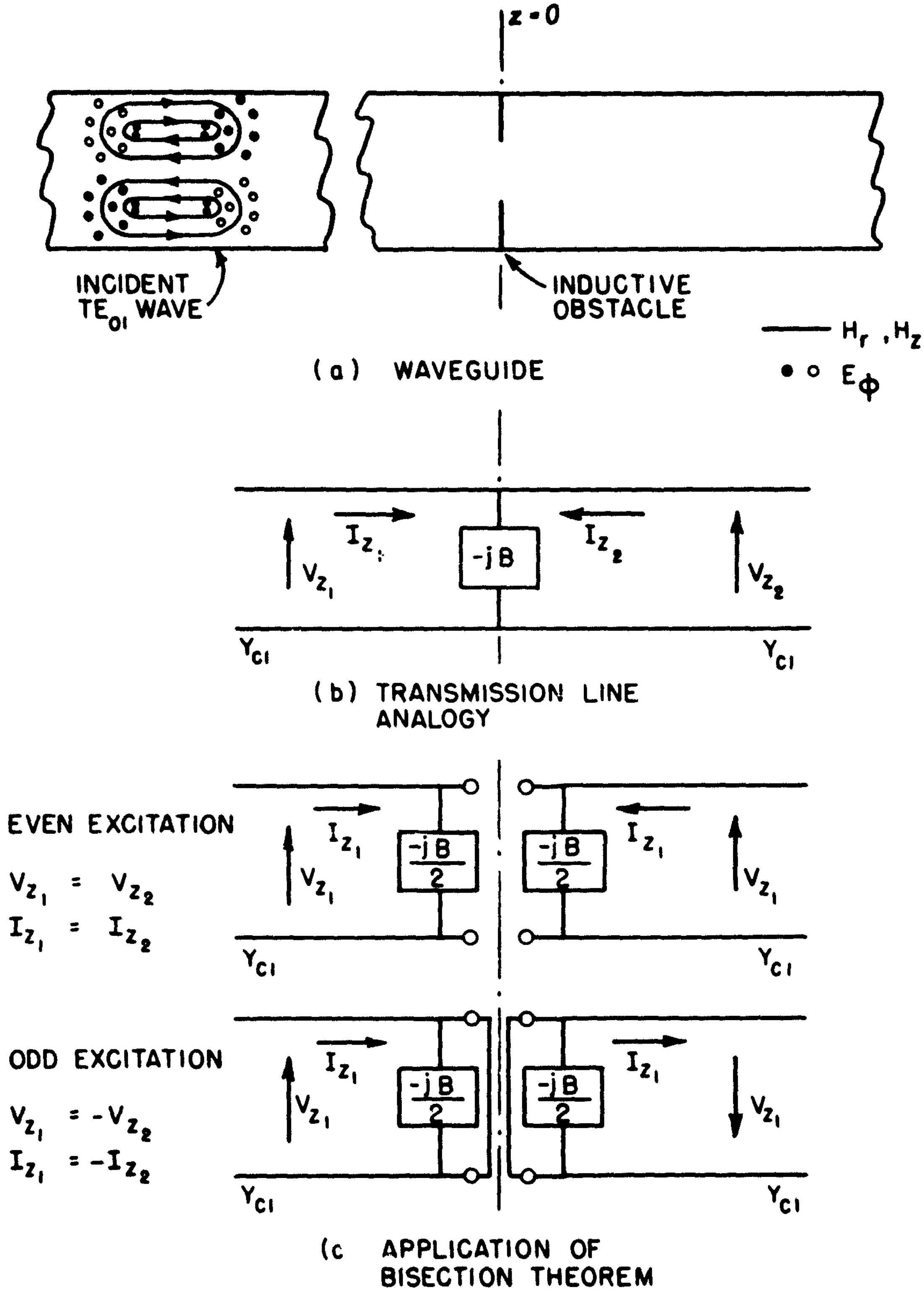


FIG. 2 SIMPLIFICATION OF WAVEGUIDE CIRCUIT

Applying the boundary condition that the transverse magnetic field must vanish on the aperture of the obstacle yields the relationship

$$I_1 h_1(r) = -\sum_{n=2}^{\infty} I_n h_n(r) \qquad \begin{matrix} z = 0 \\ 0 < r < r_o \end{matrix} \tag{9}$$

The left side of the above equation represents the propagating mode (dominant circular-electric mode) while the summation consists of higher-mode terms excited by the discontinuity of the obstacle. These higher nonpropagating modes are assumed to be completely attenuated before reaching the nearest discontinuity. This is equivalent to terminating each nonpropagating mode in its reactive characteristic admittance. Making use of the transmission-line equation relating V_n and I_n, equation (9) can be written as

$$\begin{aligned} I_1 h_1(r) &= -\sum_{n=2}^{\infty} Y_{cn} V_n h_n(r) = \\ &\sum_{n=2}^{\infty} Y_{cn} h_n(r) \int_{\text{aperture}} E_\phi(r') h_n(r') d\sigma' \end{aligned} \tag{10}$$

Interchanging the order of summation and integration yields

$$I_1 h_1(r) = \int_{\text{aperture}} G(r,r') E_\phi(r') d\sigma', \tag{11}$$

where the function $G(r,r')$ is defined as

$$G(r,r') = \sum_{n=2}^{\infty} Y_{cn} h_n(r) h_n(r') \tag{12}$$

The above equation is an integral equation with the aperture electric field as the unknown quantity. If it is possible to determine $E_\phi(r')$ from the solution of the integral equation the desired susceptance can be obtained from equation (5) and equation (8a). It then follows that the susceptance of the obstacle is

$$\frac{jB}{2} = \frac{I_1}{\int_{\text{aperture}} E_\phi(r) h_1(r) d\sigma} \tag{13}$$

Because of the inherent difficulty involved in obtaining a solution for the integral equation, an approximate expression for the obstacle susceptance will be obtained by the application of a variational principle.

Multiplying equation (10) by the unknown aperture field $E_\phi(r)$, integrating over the aperture, and dividing by V_1^2 yields the variational expression for the obstacle susceptance

$$\frac{-jB}{2} = \frac{\sum_{2}^{\infty} Y_{on}\left[\int_{\text{aperture}} E_\phi(r) h_n(r) d\sigma\right]^2}{\left[\int_{\text{aperture}} E_\phi(r) h_1(r) d\sigma\right]^2} \tag{14}$$

It can be readily shown that this expression for the obstacle susceptance is stationary with respect to small variations of $E_\phi(r)$ about the correct aperture electric field. If the chosen trial function is correct to the first order, the resultant susceptance will be correct to the second order. If the chosen trial field is the actual aperture electric field, then the calculated susceptance is a minimum. This expression is particularly convenient because the results are independent of the amplitude of the trial function.

The susceptance determined by the above method is an upper bound to the correct susceptance. A lower bound can be derived from a similar formulation involving an assumed current distribution on the obstacle. The lower-bound susceptance obtained in this manner is

$$-\frac{jB}{2} = \frac{\left[\int_{\text{obstacle}} H_r(r)e_1(r)d\sigma\right]^2}{\sum_2^{\infty} Z_{cn}\left[\int_{\text{obstacle}} H_r(r)e_n(r)d\sigma\right]^2} \qquad (15)$$

II

Lower-Bound Expression for the Obstacle Susceptance

The assumed obstacle current (the tangential component of the magnetic field) is chosen to be the obstacle current as the aperture tends to zero and a correction term. The trial field chosen, consisting of the first two terms of a set of orthogonal functions, is

$$H_r(r) = J_1(\rho_1 \frac{r}{a}) + AJ_1(\rho_2 \frac{r}{a}), \quad r_o \leq r \leq a, \qquad (16)$$

where A is a constant to be evaluated later.

It would be possible to obtain a very accurate solution by assuming as a trial function the sum of a large number of orthogonal functions with undetermined coefficients. The calculated results obtained by determining the unknown coefficients so as to make B stationary would yield an accurate value for the susceptance of the obstacle. Practically, however, the calculations become extremely tedious for trial functions consisting of more than two terms. For the practical evaluation of the obstacle susceptance, the analysis of the lower- and upper-bound expressions is specialized to trial functions containing only two terms.

The assumed field $H_r(r)$ is inserted in equation (15) and the variational expression for the obstacle susceptance is written in the following convenient form

$$\frac{B_a}{Y_{c1}\lambda_g} = \frac{\left[\int_{r_o}^{a} J_1(\rho_1 \frac{r}{a})\left\{J_1(\rho_1 \frac{r}{a}) + A\, J_1(\rho_2 \frac{r}{a})\right\} rdr\right]^2}{\pi J_o^2(\rho_1) \sum_{n=2}^{\infty} \frac{\left[\int_{r_o}^{a} J_1(\rho_n \frac{r}{a}) \left\{J_1 \rho_1 \frac{r}{a}) + A\, J_1(\rho_2 \frac{r}{a})\right\} rdr\right]^2}{\sqrt{\rho_n^2-(ka)^2} \qquad J_o^2(\rho_n)}} \tag{17}$$

The integrals in the previous equation can be evaluated by using these standard forms for the Bessel functions:[20]

$$\int_{r_o}^{a} J_1(\rho_m \frac{r}{a}) J_1(\rho_n \frac{r}{a}) rdr \qquad m \neq n$$

$$= \frac{a^2\delta}{\rho_m^2-\rho_n^2} \left[\rho_m J_o(\rho_m\delta) J_1(\rho_n\delta) - \rho_n\, J_1(\rho_m\delta) J_o(\rho_n\delta)\right] \tag{18}$$

and

$$\int_{r_o}^{a} J_1(\rho_m \frac{r}{a})^2 rdr$$

$$= \frac{a^2\delta}{2} \left[\delta\left\{J_o(\rho_m\delta) J_2(\rho_m\delta) - J_1^2(\rho_m\delta)\right\} - \frac{J_o(\rho_m) J_2(\rho_m)}{\delta}\right] \tag{19}$$

Equation (17) can be simplified by letting

$$a^2\delta\left\{G_1 + A\, G_2\right\} = \int_{r_o}^{a} J_1(\rho_n \frac{r}{a}) J_1(\rho_1 \frac{r}{a}) rdr + A \int_{r_o}^{a} J_1(\rho_n \frac{r}{a}) J_1(\rho_2 \frac{r}{a}) rdr \tag{20}$$

and

$$a^2\delta\left\{F_1 + AF_2\right\} = \int_{r_o}^{a} \left[J_1(\rho_1 \frac{r}{a})\right]^2 rdr + A \int_{r_o}^{a} J_1(\rho_1 \frac{r}{a}) J_1(\rho_2 \frac{r}{a}) rdr \tag{21}$$

Performing the required integrations results in

$$G_1 = \frac{1}{\rho_n^2-\rho_1^2}\left[\rho_n J_o(\rho_n\delta)J_1(\rho_1\delta)-\rho_1 J_1(\rho_n\delta)J_o(\rho_1\delta)\right] \quad n \geq 2 \tag{22}$$

$$\begin{cases} G_2 = \frac{1}{2}\left[\delta\left\{J_o(\rho_2\delta)J_2(\rho_2\delta)-J_1^2(\rho_2\delta)\right\} - \frac{J_o(\rho_2)J_2(\rho_2)}{\delta}\right] & n = 2 \\ G_2 = \frac{1}{\rho_n^2-\rho_2^2}\left[\rho_n J_o(\rho_n\delta)J_1(\rho_2\delta)-\rho_2 J_1(\rho_n\delta)J_o(\rho_2\delta)\right] & n > 2 \end{cases} \tag{23}$$

$$F_1 = \frac{1}{2}\left[\delta\left\{J_o(\rho_1\delta)J_2(\rho_1\delta)-J_1^2(\rho_1\delta)\right\} - \frac{J_o(\rho_1)J_2(\rho_1)}{\delta}\right] \tag{24}$$

$$F_2 = \frac{1}{\rho_2^2-\rho_1^2}\left[\rho_2 J_o(\rho_2\delta)J_1(\rho_1\delta)-\rho_1 J_1(\rho_2\delta)J_o(\rho_1\delta)\right] \tag{25}$$

Substitution of the above expressions into the equation for the obstacle susceptance yields

$$\frac{Ba}{Y_{c1}\lambda_g} = \frac{[1+A']^2}{\pi J_o^2(\rho_1)\sum\limits_2^{\infty}\frac{\left[\frac{G_1}{F_1}+A'\frac{G_2}{F_2}\right]^2}{\sqrt{\rho_n^2-(ka)^2}\,J_o^2(\rho_n)}} \tag{26}$$

where

$$A' = A\frac{F_2}{F_1} \tag{27}$$

is a constant to be evaluated.

A further simplification is achieved by defining

$$S_G = \pi J_o^2(\rho_1)\sum_2^{\infty}\frac{[G_1/F_1]^2}{\sqrt{\rho_n^2-(ka)^2}\,J_o^2(\rho_n)} \tag{28}$$

$$S_1 = \pi J_o^2(\rho_1)\sum_2^{\infty}\frac{[G_1G_2/F_1F_2]}{\sqrt{\rho_n^2-(ka)^2}\,J_o^2(\rho_n)} \tag{29}$$

$$S_2 = \pi J_o^2(\rho_1) \sum_2^\infty \frac{[Q_2/P_2]^2}{\sqrt{\rho_n^2-(ka)^2}\, J_o^2(\rho_n)} \tag{30}$$

The normalized susceptance for the circular obstacle can be simply expressed as

$$\frac{Ba}{Y_{cl}\lambda_g} = \frac{[1+A']^2}{S_o + 2A'S_1 + A'^2 S_2} \tag{31}$$

The condition for the susceptance to be stationary with respect to the arbitrary coefficient A' is that the derivative of B with respect to A' must be zero. Performing the differentiation and setting the result equal to zero, one obtains

$$A' = \frac{S_1 - S_o}{S_1 - S_2} \tag{32}$$

Inserting the value for A' into equation (31) gives

$$\frac{Ba}{Y_{cl}\lambda_g} = \frac{1}{S_o - \dfrac{(S_1 - S_o)^2}{(S_o + S_2 - 2S_1)}} \qquad \text{First order} \tag{33}$$

for the two-term trial function. The susceptance using a one-term trial function ($A' = 0$) is given by

$$\frac{Ba}{Y_{cl}\lambda_g} = \frac{1}{S_o} \qquad \text{Zero order} \tag{34}$$

It can readily be shown that

$$\frac{(S_1 - S_o)^2}{(S_o + S_2 - 2S_1)}$$

in the denominator of equation (33) is a positive quantity; therefore the susceptance determined by insertion of the two-term trial function is larger than the susceptance determined from the first term.

Referring to the trial function of equation (16), the coefficient A is evaluated to give the minimum susceptance for the chosen trial function, yielding

$$H_r(r) = J_1(\rho_1 \tfrac{r}{a}) + \left[\frac{S_1 - S_0}{S_1 - S_2}\right] \frac{F_1}{F_2} J_1(\rho_2 \tfrac{r}{a}) \qquad (35)$$

III

Upper-Bound Expression for the Obstacle Susceptance

Two different trial functions are used in the upper-bound variational formulation for the normalized susceptance of the circular obstacle.

The first trial function consists of the unperturbed aperture electric field and a correction term. This assumed field which will give accurate values for the susceptance of obstacles with large apertures (i.e., small perturbation of the incident field) is

$$E_\phi(r) = J_1(\rho_1 \tfrac{r}{r_o}) + C\, J_1(\rho_2 \tfrac{r}{r_o}) \qquad 0 \le r \le r_o \qquad (36)$$

The above function satisfies the boundary condition that the electric field must vanish at the axis and at $r = r_o$.

Insertion of the trial function into the upper-bound expression for the obstacle susceptance allows the variational form for the susceptance to be expressed as

$$\frac{B_a}{Y_{cl}\lambda_g} = \frac{J_o^2(\rho_1) \displaystyle\sum_{n=2}^{\infty} \sqrt{\rho_n^2-(ka)^2}\left[\int_0^{r_o} J_1(\rho_n \tfrac{r}{a}) \left\{ J_1(\rho_1 \tfrac{r}{r_o}) + CJ_1(\rho_2 \tfrac{r}{r_o})\right\} r\,dr\right]^2}{\pi \left[\displaystyle\int_0^{r_o} J_1(\rho_1 \tfrac{r}{a})\left\{J_1(\rho_1 \tfrac{r}{r_o}) + CJ_1(\rho_2 \tfrac{r}{r_o})\right\} r\,dr\right]^2} \qquad (37)$$

The integrations are carried out by Lommel's Integral Formulae (as in equations 18 and 19). The relative susceptance can be simply expressed as

$$\frac{Ba}{Y_{el}\lambda_g} = \frac{T_o + 2C'T_1 + C'^2T_2}{(1 + C')^2}, \tag{38}$$

where

$$T_K = \frac{(\delta^2-1)^{(2-K)}(\delta^2-\tau^2)^K J_o^2(\rho_1)}{\pi \, J_1^2(\rho_1\delta)} \sum_{n=2}^{\infty} \frac{\sqrt{\rho_n^2-(ka)^2}\; J_1^2(\rho_n\delta)}{J_o^2(\rho_n)(\alpha_n^2-1)^{2-K}(\alpha_n^2-\tau^2)^K}$$

$$K = 0,\ 1,\ 2 \tag{39}$$

and the following quantities are defined:

$$C' = \frac{C\, J_o(\rho_2)}{\tau J_o(\rho_1)} \tag{40}$$

$$\tau = \frac{\rho_2}{\rho_1} \tag{41}$$

$$\alpha_n = \frac{\rho_n}{\rho_1}\delta \tag{42}$$

The coefficient C' which yields the minimum susceptance for the chosen trial function is given by

$$C' = \frac{T_1 - T_o}{T_1 - T_2} \tag{43}$$

Substituting the above quantity into equation (38) gives

$$\frac{Ba}{Y_{el}\lambda_g} = T_o - \frac{(T_1 - T_o)^2}{T_o + T_2 - 2T_1} \quad \text{First order} \tag{44}$$

for the assumed two-term trial function.

The inductive susceptance determined by using the unperturbed field $E_\phi(r) = J_1(\rho_1 \frac{r}{r_o})$ as a trial function is given by

$$\frac{Ba}{Y_{el}\lambda_g} = T_o \quad \text{Zero order} \tag{45}$$

It can furthermore be shown that $(T_1-T_o)^2/(T_o+T_2-2T_1)$ is always a positive quantity indicating that the first-order susceptance of equation (44) is less than the zero-order susceptance of equation (45).

The trial function of equation (36) with C evaluated to give the minimum susceptance for the choice of trial function then becomes

$$E_\phi(r) = J_1(\rho_1\sigma) + \left[\frac{T_1 - T_o}{T_1 - T_2}\right]\tau \frac{J_o(\rho_1)}{J_o(\rho_2)} J_1(\rho_2\sigma) \qquad (46)$$

where $\sigma = \frac{r}{r_o}$.

To this point trial functions have been considered that are expected to give accurate values for the susceptance of obstacles with very small or very large apertures. The final trial function to be inserted in the upper-bound expression of equation (14) is an approximation to the aperture electric field determined from the quasi-static solution. The particular function selected is chosen so that the integrals in the variational expression can be readily evaluated.

The trial function which meets these requirements and satisfies the boundary conditions is

$$E_\phi(r) = r\sqrt{r_o^2 - r^2} + D\, r^3\sqrt{r_o^2 - r^2} \qquad (47)$$

Inserting this function into the variational form for the relative susceptance results in

$$\frac{B_a}{Y_{c1}\lambda_g} = \frac{J_o^2(\rho_1)\sum_{n=2}^{\infty}\sqrt{\rho_n^2-(ka)^2}\left[\int_0^{r_o} J_1(\rho_n \frac{r}{a})\left\{r^2\sqrt{r_o^2-r^2}\left[1+Dr^2\right]\right\}dr\right]^2}{\pi\left[\int_0^{r_o} J_1(\rho_1 \frac{r}{a})\left\{r^2\sqrt{r_o^2-r^2}\left[1+Dr^2\right]\right\}dr\right]^2} \qquad (48)$$

The integrals to be evaluated are of the form

$$I = \int_0^{r_o} J_1(\rho_n \tfrac{r}{a})\, r^m \sqrt{r_o^2 - r^2}\, dr \tag{49}$$

These integrals can be evaluated by making the substitution $r = r_o \sin\theta$; it then follows that

$$I = r_o^{(m+2)} \int_0^{\pi/2} J_1(\rho_n \delta \sin\theta) \sin^m\theta \cos^2\theta \, d\theta \tag{50}$$

This integral is evaluated from Sonine's first integral formula [21], yielding

$$I = r_o^{(m+2)} \int_0^{\pi/2} J_1(\rho_n \delta \sin\theta) \sin^m\theta \cos^2\theta \, d\theta$$

$$= \frac{\sqrt{2}\; r_o^{(m+2)} \Gamma(3/2)}{(\rho_n \delta)^{3/2}} J_{(m+1/2)} (\rho_n \delta) \tag{51}$$

where $\Gamma(x)$ is the gamma-function of x.

Performing the integrations in equation (48) the expression for the normalized susceptance becomes

$$\frac{Ba}{Y_{cl}\lambda_g} = \frac{U_o + 2D'U_1 + D'^2U_2}{(1 + D')^2}\,, \tag{52}$$

where D' is an arbitrary constant defined by

$$D' = D\, r_o^2 \frac{J_{9/2}(\rho_1\delta)}{J_{5/2}(\rho_1\delta)} \qquad \text{and} \tag{53}$$

$$U_K = \frac{J_o^2(\rho_1)\, (\rho_1)^3 \left[J_{5/2}(\rho_1\delta)\right]^K}{\pi \left[J_{9/2}(\rho_1\delta)\right]^K} \sum_2^{\infty} \frac{\sqrt{\rho_n^2 - (ka)^2}\left[J_{5/2}(\rho_n\delta)\right]^{2-K}}{J_o^2(\rho_n)\, \rho_n^3 \left[J_{5/2}(\rho_1\delta)\right]^{2-K}}$$

$$K = 0,\ 1,\ 2 \tag{54}$$

Proceeding as in the previous two cases, the arbitrary coefficient is determined by differentiating equation (52) with respect to D' and by setting the resultant expression equal to zero. The constant is therefore evaluated to be

$$D' = \frac{U_1 - U_o}{U_1 - U_2} \tag{55}$$

It is now possible to express the first-order susceptance as

$$\frac{Ba}{Y_{cl}\lambda_g} = U_o - \frac{(U_1 - U_o)^2}{U_o + U_2 - 2U_1} \tag{56}$$

and the zero-order susceptance as

$$\frac{Ba}{Y_{cl}\lambda_g} = U_o \quad . \tag{57}$$

The trial function of equation (47) with the coefficient D adjusted to yield a minimum susceptance for this choice of trial function, becomes

$$\frac{E_g(r)}{r_o^2} = \left\{ 1 + \left[\frac{U_1 - U_o}{U_1 - U_2} \right] \frac{J_{5/2}(\rho_1 \delta)}{J_{9/2}(\rho_1 \delta)} \sigma^2 \right\} \sigma \sqrt{1 - \sigma^2} \tag{58}$$

IV

Theoretical and Experimental Results

The inductive susceptance of the circular obstacle was calculated for values of δ between .5 and 1. Smaller values of δ are of little interest because little energy is transmitted through small apertures. Calculations of the lower-bound expression for the susceptance (equation (35) as the trial function) and the upper-bound expression for the susceptance (equations (46) and (58) used as trial functions) were carried out for $\frac{a}{\lambda} = .75$. The zero-order and first-order susceptances and also the experimental results are plotted on Figs. 3, 4, and 5. Table I consists of the experimental and theoretical data used to construct these curves. It is observed that the first-order susceptance determined by the lower-bound variational formulation and the first-order susceptance derived from the assumed aperture electric field of equation (46) in the upper-bound formulation are in excellent agreement with the experimental results. The difference between the zero- and the first-order susceptances is a convenient measure of the closeness of the assumed field to the correct field.

It is of particular interest to observe the manner in which a particular trial function varies with the size of the obstacle aperture. In Figs. 6, 7, and 8 the trial functions are plotted as a function of radial position with δ as a parameter. Of course it is impossible to find an exact correlation between the assumed fields and the correct obstacle susceptance because the actual aperture field and obstacle current are not known.

The first-order susceptance determined by inserting $E_\phi(r)$ of equation (46) in the upper-bound variational expression for the obstacle susceptance is plotted in Fig. 9 for values of $\frac{a}{\lambda}$ equal to .70, .75, .90, 1.0. These theoretical values are compared with the experimental values and are in excellent agreement for values of δ greater than .55. The curves of Fig. 9 can be used to obtain the susceptance of a circular obstacle providing the obstacle structure meets the requirements set forth by the theory.

V

Experimental Discussion

The susceptance of the circular obstacle was measured by the resonance-curve method. An excellent description of the experimental technique used for the precise measurement of waveguide discontinuities is given by Huxley.[22]

The circular waveguide, in which the susceptance of the circular obstacle was measured, consists of brass tubing with 5.750-in. i.d. and .125-in. wall. At a frequency of 3000 Mc/s the waveguide is capable of supporting six propagating modes

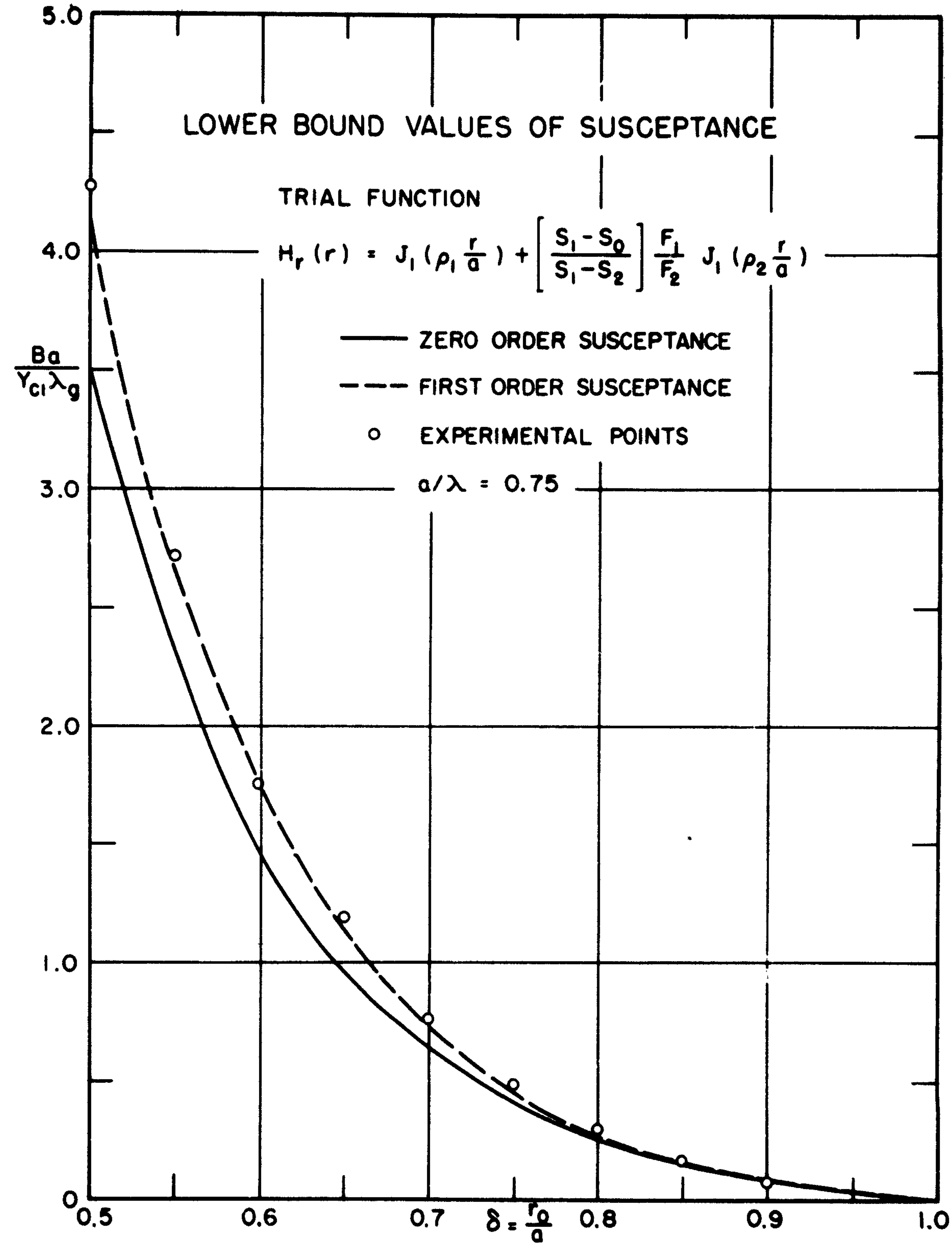
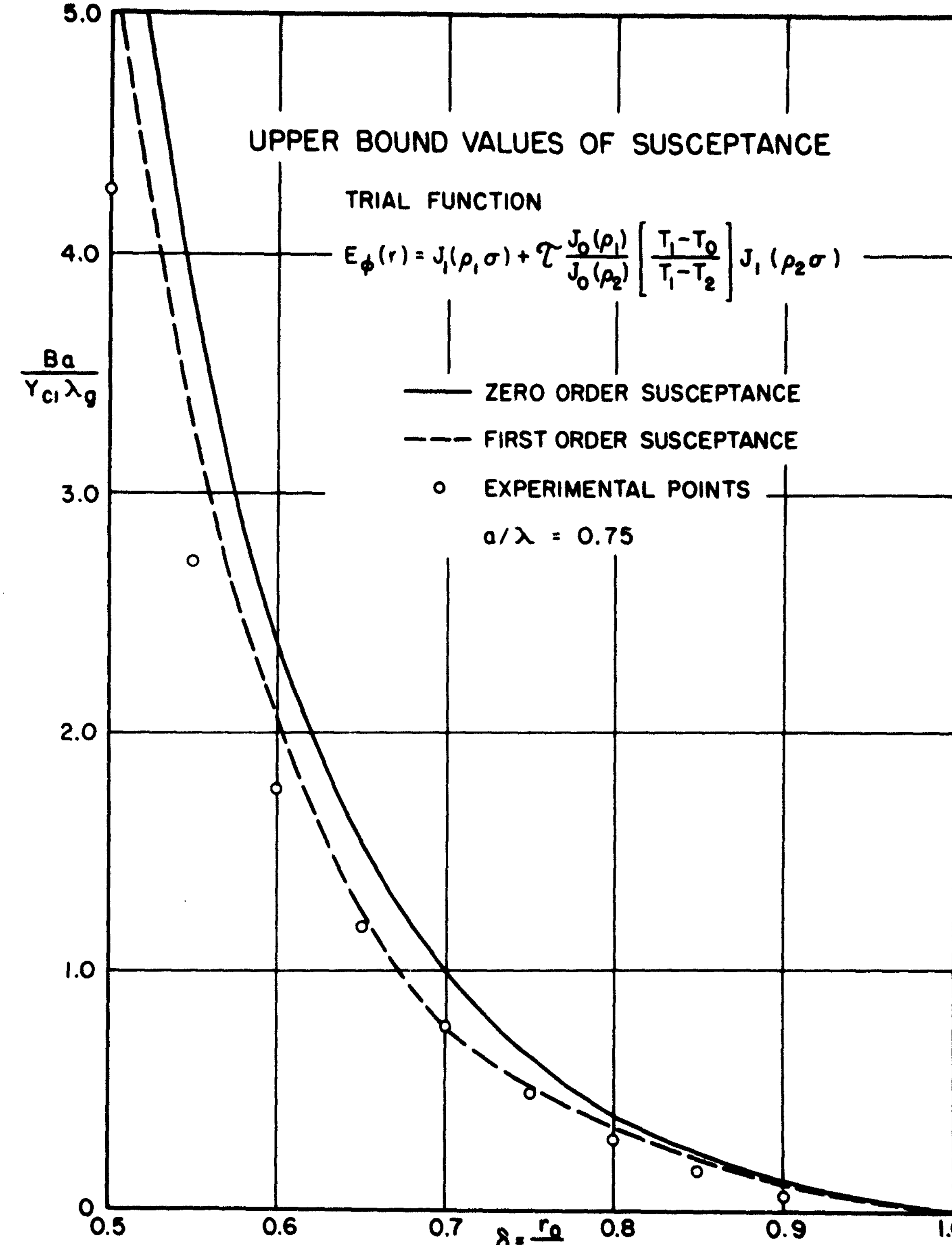

LOWER BOUND VALUES OF SUSCEPTANCE
TRIAL FUNCTION
$H_r(r) = J_1(\rho_1 \frac{r}{a}) + \left[\frac{S_1 - S_0}{S_1 - S_2}\right] \frac{F_1}{F_2} J_1(\rho_2 \frac{r}{a})$
ZERO ORDER SUSCEPTANCE
FIRST ORDER SUSCEPTANCE
EXPERIMENTAL POINTS
$a/\lambda = 0.75$
$\frac{Ba}{Y_{c1}\lambda_g}$
5.0
4.0
3.0
2.0
1.0
0
0.5
0.6
0.7
$\delta = \frac{r_0}{a}$
0.8
0.9
1.0

FIGURE 3



UPPER BOUND VALUES OF SUSCEPTANCE
TRIAL FUNCTION
$E_\phi(r) = J_1(\rho_1\sigma) + \tau \frac{J_0(\rho_1)}{J_0(\rho_2)}\left[\frac{T_1-T_0}{T_1-T_2}\right] J_1(\rho_2\sigma)$
ZERO ORDER SUSCEPTANCE
FIRST ORDER SUSCEPTANCE
EXPERIMENTAL POINTS
$a/\lambda = 0.75$
$\frac{Ba}{Y_{C1}\lambda_g}$
5.0
4.0
3.0
2.0
1.0
0
0.5
0.6
0.7
0.8
0.9
1.0
$\delta = \frac{r_0}{a}$

FIGURE 4

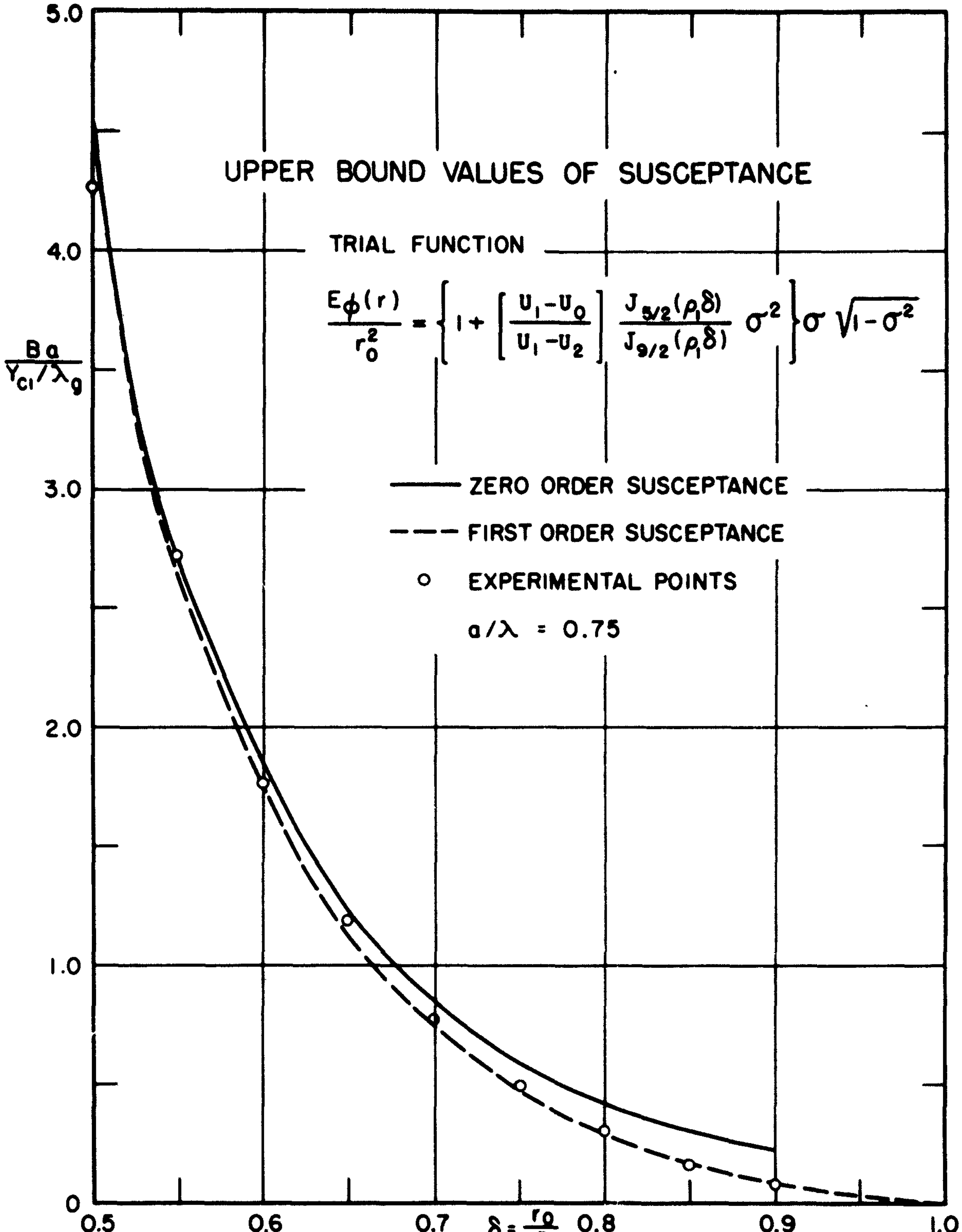

UPPER BOUND VALUES OF SUSCEPTANCE
TRIAL FUNCTION
$\frac{E_\phi(r)}{r_0^2} = \left\{1 + \left[\frac{U_1 - U_0}{U_1 - U_2}\right] \frac{J_{5/2}(\rho_1\delta)}{J_{9/2}(\rho_1\delta)} \sigma^2\right\} \sigma\sqrt{1-\sigma^2}$
ZERO ORDER SUSCEPTANCE
FIRST ORDER SUSCEPTANCE
EXPERIMENTAL POINTS
$a/\lambda = 0.75$
$\frac{Ba}{Y_{c1}/\lambda_g}$
$\delta = \frac{r_0}{a}$
5.0
4.0
3.0
2.0
1.0
0
0.5
0.6
0.7
0.8
0.9
1.0

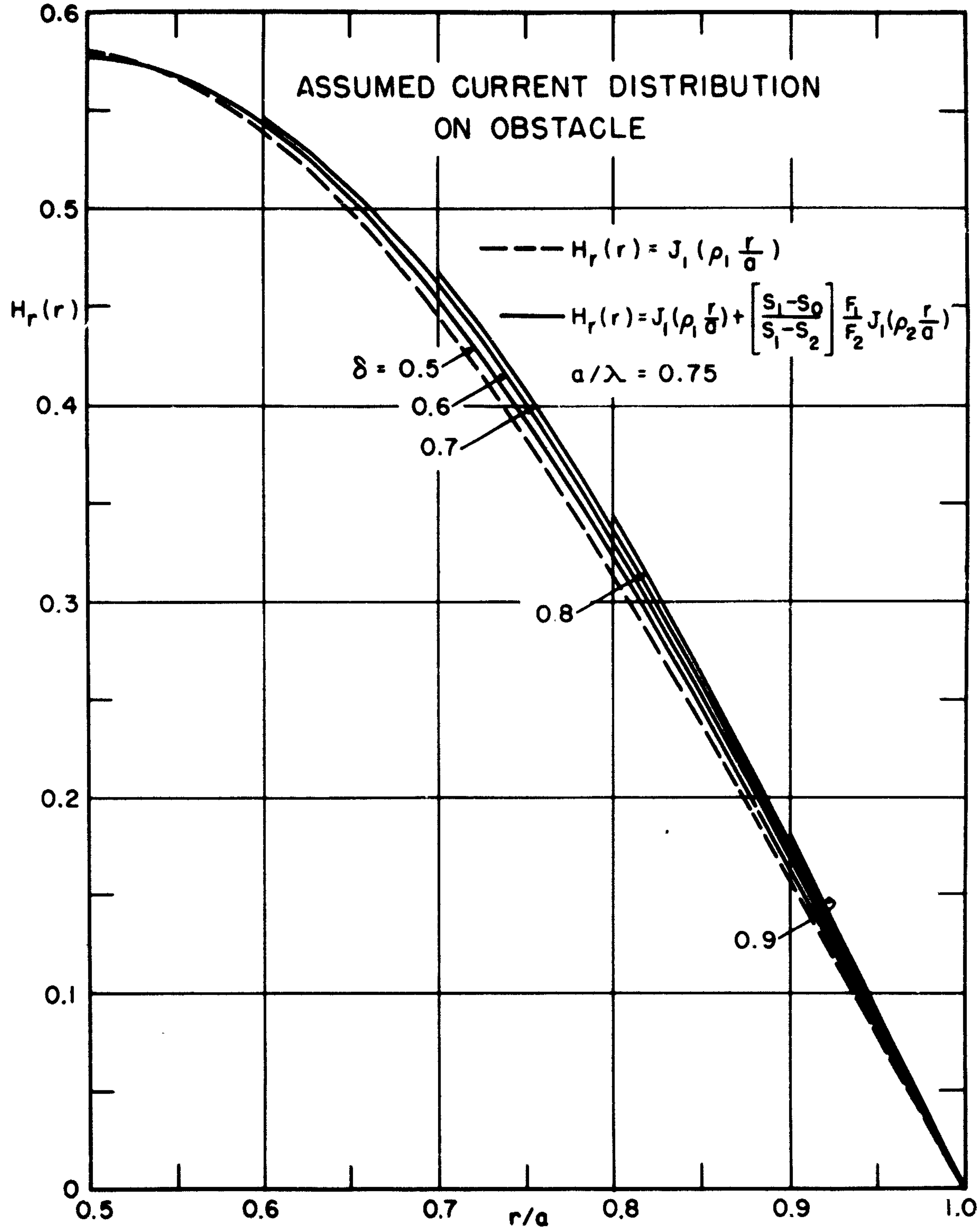
FIGURE 5

Table I

$\frac{a}{\lambda} = .75$

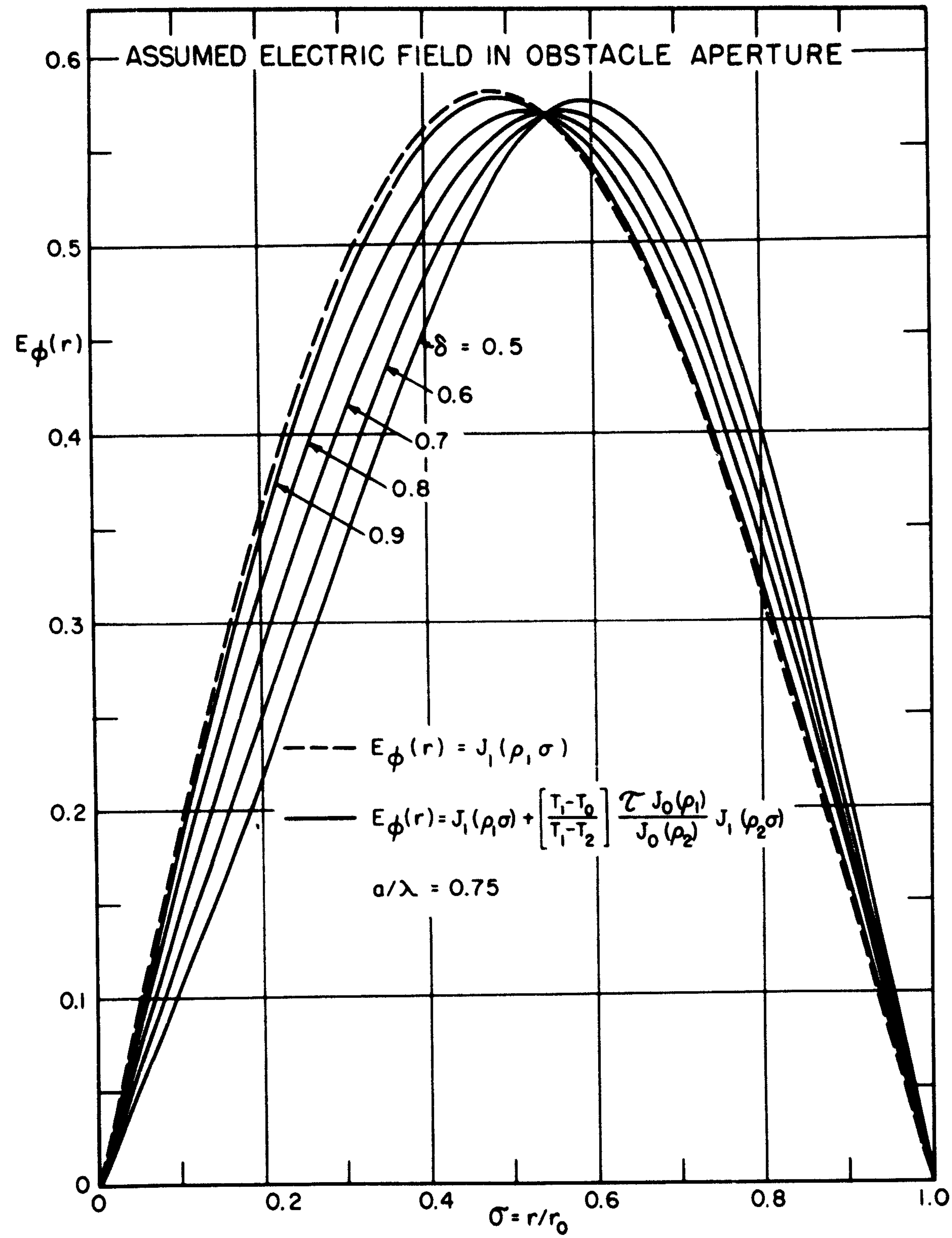
Lower Bound — Upper Bound

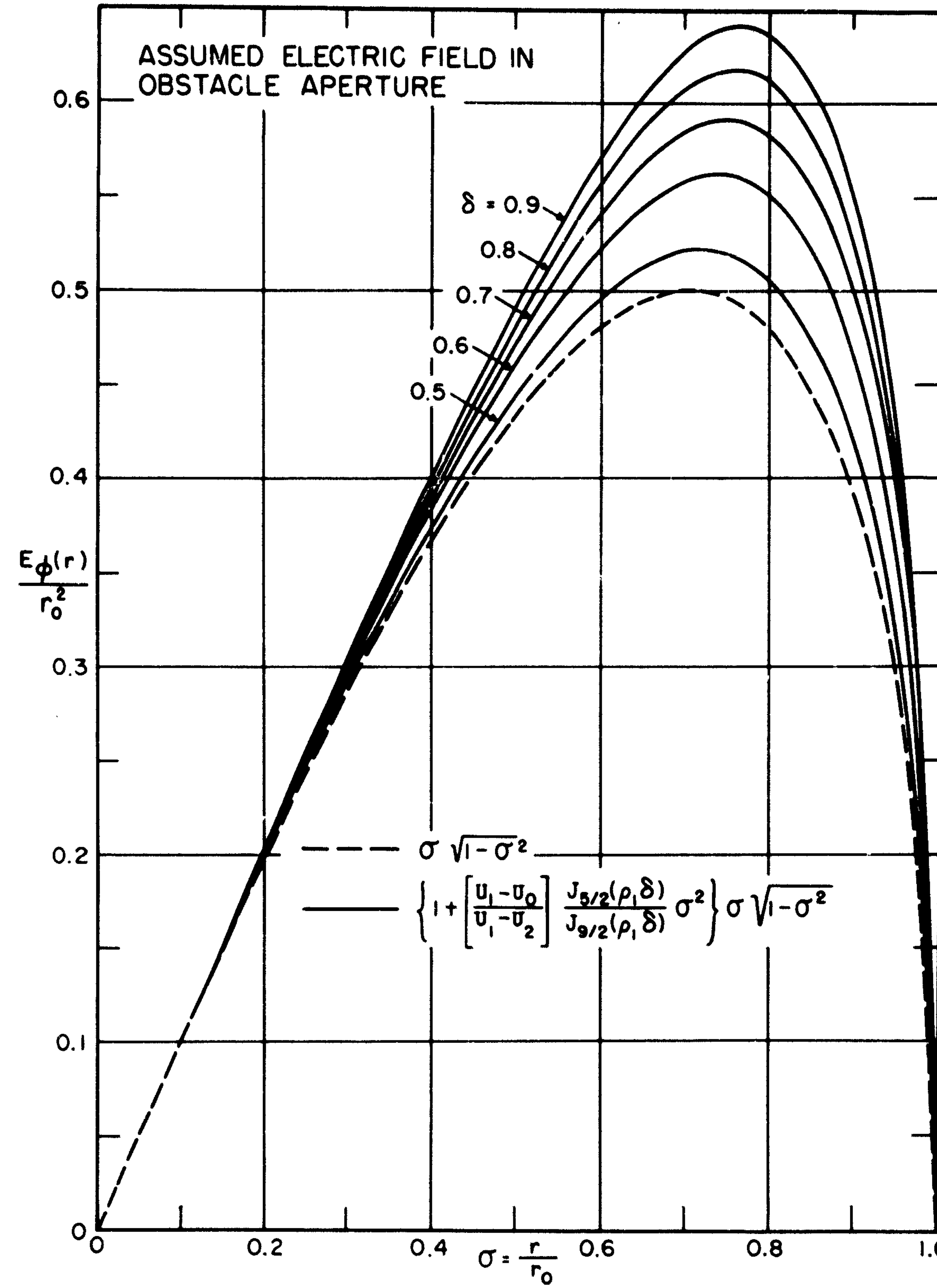
δ	$\frac{1}{S_o}$	$\frac{1}{S_o - \frac{(S_1-S_o)^2}{S_o+S_2-2S_1}}$	T_o	$T_o - \frac{(T_1-T_2)^2}{T_o+T_2-2T_1}$	U_o	$U_o - \frac{(U_1-U_o)^2}{U_o+U_2-2U_1}$	Experimental Values
.50	3.49	4.13	6.21	5.40	4.52	4.48	4.27
.55	2.33	2.66	3.60	3.10	2.72	2.66	2.71
.60	1.47	1.75	2.41	2.11	1.86	1.77	1.76
.65	.958	1.12	1.51	1.33	1.23	1.12	1.19
.70	.649	.739	1.01	.890	.867	.754	.767
.75	.415	.450	.632	.561	.592	.472	.490
.80	.259	.263	.398	.353	.417	.290	.298
.85	.142	.142	.222	.201	.306	.161	.163
.90	.068	.071	.109	.103	.223	.081	.070



ASSUMED CURRENT DISTRIBUTION ON OBSTACLE
$H_r(r) = J_1(\rho_1 \frac{r}{a})$
$H_r(r) = J_1(\rho_1 \frac{r}{a}) + \left[\frac{S_1 - S_0}{S_1 - S_2}\right] \frac{F_1}{F_2} J_1(\rho_2 \frac{r}{a})$
$a/\lambda = 0.75$
$\delta = 0.5$
0.6
0.7
0.8
0.9
$H_r(r)$
r/a
0.6
0.5
0.4
0.3
0.2
0.1
0
0.5
0.6
0.7
0.8
0.9
1.0

FIGURE 6


ASSUMED ELECTRIC FIELD IN OBSTACLE APERTURE
0.6
0.5
0.4
0.3
0.2
0.1
0
$E_\phi(r)$
δ = 0.5
0.6
0.7
0.8
0.9
$E_\phi(r) = J_1(\rho_1\sigma)$
$E_\phi(r) = J_1(\rho_1\sigma) + \left[\frac{T_1-T_0}{T_1-T_2}\right] \frac{\tau J_0(\rho_1)}{J_0(\rho_2)} J_1(\rho_2\sigma)$
a/λ = 0.75
0
0.2
0.4
0.6
0.8
1.0
$\sigma = r/r_0$
FIGURE 7



ASSUMED ELECTRIC FIELD IN OBSTACLE APERTURE
δ = 0.9
0.8
0.7
0.6
0.5
0.6
0.5
0.4
0.3
0.2
0.1
0
$\frac{E_\phi(r)}{r_0^2}$
0
0.2
0.4
0.6
0.8
1.0
$\sigma = \frac{r}{r_0}$
$\sigma\sqrt{1-\sigma^2}$
$\left\{1+\left[\frac{U_1-U_0}{U_1-U_2}\right]\frac{J_{5/2}(\rho_1\delta)}{J_{9/2}(\rho_1\delta)}\sigma^2\right\}\sigma\sqrt{1-\sigma^2}$

FIGURE 8

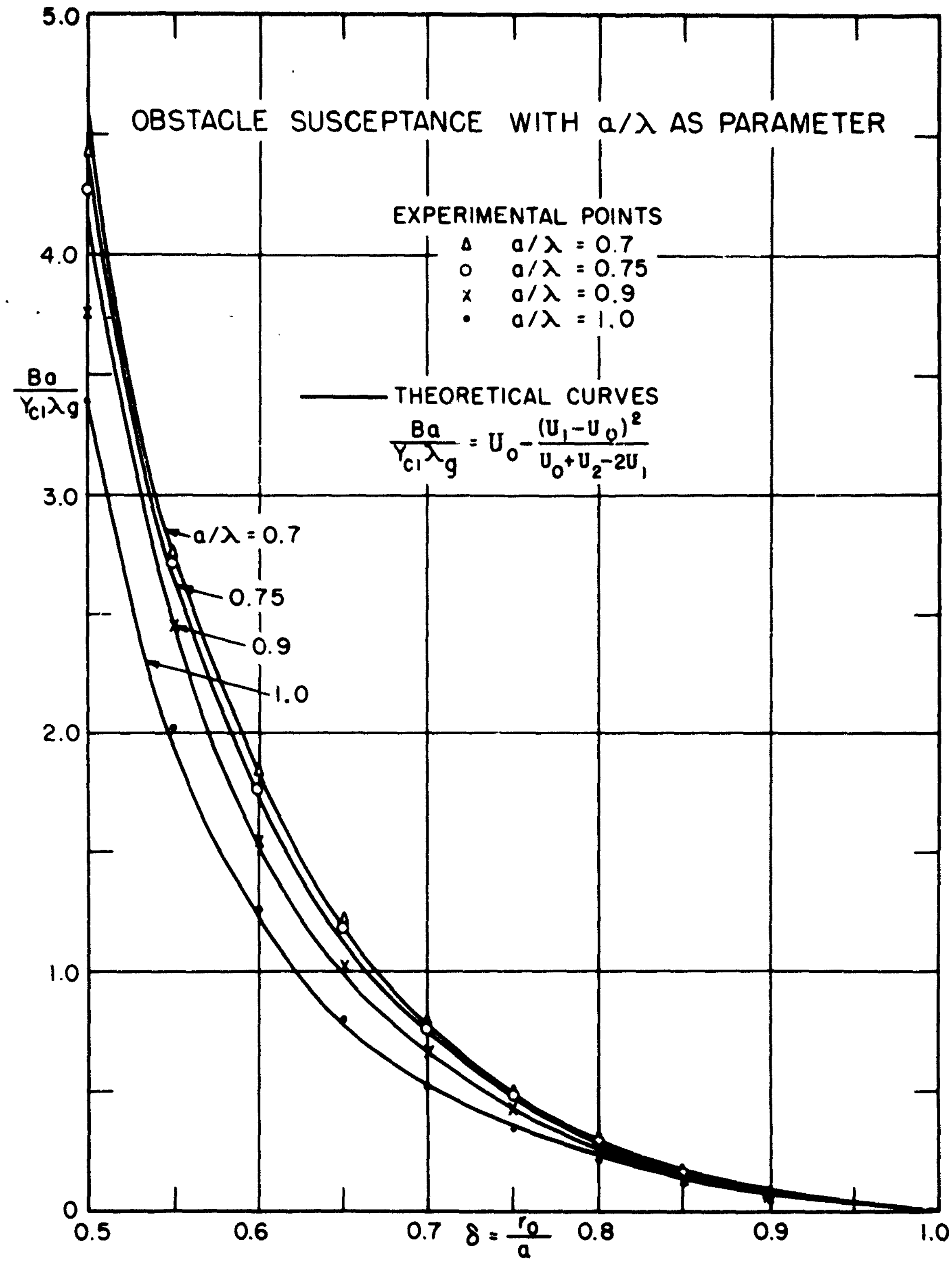

OBSTACLE SUSCEPTANCE WITH a/λ AS PARAMETER
EXPERIMENTAL POINTS
Δ $a/\lambda = 0.7$
o $a/\lambda = 0.75$
x $a/\lambda = 0.9$
• $a/\lambda = 1.0$
THEORETICAL CURVES
$\frac{Ba}{Y_{c1}\lambda_g} = U_0 - \frac{(U_1 - U_0)^2}{U_0 + U_2 - 2U_1}$
$a/\lambda = 0.7$
0.75
0.9
1.0
$\frac{Ba}{Y_{c1}\lambda_g}$
5.0
4.0
3.0
2.0
1.0
0
0.5
0.6
0.7
0.8
0.9
1.0
$\delta = \frac{r_0}{a}$

FIGURE 9

(TE_{11}, TM_{01}, TE_{21}, TE_{01}, TM_{11}, TE_{31}).

The generator of the "TE_{01} transmission line" consists of a shielded loop of 3/16 in. diameter, constructed from 1/32 in. o.d. coaxial line. The generator is mounted on the face of one of the two noncontacting short-circuiting plungers. The loop is oriented to couple maximum energy to the dominant circular-electric mode.

The detector consists of four identical shielded loops similar to the excitation loop, and uniformly spaced about the circumference of the waveguide. The plane of the detecting loops is oriented in the transverse plane and adjusted so that the currents excited on the loops are codirectional. Individual detector loop outputs of equal phase and amplitude are connected together and to the radio-frequency receiver.

Owing to the discriminating properties of the detector and the damping effect of the lossy material behind each noncontacting short-circuiting plunger, the TE_{01} mode was observed to be approximately 60 db above extraneous modes over a free-space-wavelength range from 7.30 to 11.90 cm.

The circular obstacles were fabricated from brass flat stock of .016-in. thickness. The obstacle thickness t in terms of guide wavelength is given in Table II below.

<u>Table II</u>

$\frac{a}{\lambda}$	λ_{cm} calculated	$\lambda_{g\,cm}$ measured	t/λ_g
.70	10.441	21.262	.00191
.75	9.737	16.700	.00243
.90	8.150	11.110	.00366
1.00	7.321	9.245	.00440

It is estimated that the susceptances were measured with an order of accuracy of .1 per cent.

VI

Conclusions

Lower-bound and upper-bound expressions have been obtained for the susceptance of a circular obstacle excited by the TE_{01} mode in circular waveguide. The assumed fields used in the variational formulations are plotted to give a qualitative description of the actual fields that exist on the obstacle. The discrepancy between experimental and theoretical results is probably due to:

1. Approximate nature of the theoretical values to the true values of susceptance.
2. The finite thickness of the obstacle.
3. Finite conductivity of the obstacle.
4. Ellipticity of the circular waveguide.
5. Presence of lower-order propagating modes.
6. Difficulty in keeping the thin obstacle planar.
7. Slight asymmetry of the obstacle aperture with respect to the guide axis.
8. Typical errors involved in the measurement of admittance by the resonance-curve method.

Acknowledgment

The author wishes to thank Dr. James E. Storer for his reading and helpful criticism of this paper. The lengthy numerical computations were carried out by Miss Lesley Gerould and Miss Mary Tynan.

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